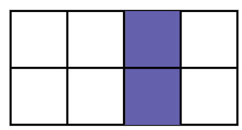
Accuracy Round

Lexington High School

December 11th, 2021

1. **[6]** Sam writes three 3-digit positive integers (that don't end in 0) on the board and adds them together. Jessica reverses each of the integers, and adds the reversals together. (For example, $\overline{XYZ}$ becomes $\overline{ZYX}$.) What is the smallest possible positive three-digit difference between Sam's sum and Jessica's sum?

2. **[8]** A random rectangle (not necessarily a square) with positive integer dimensions is selected from the 2×4 grid below. The probability that the selected rectangle contains only white squares can be written as $\frac{a}{b}$ where a and b are relatively prime positive integers. Find $a+b$.



3. **[10]** Two circles with radius 2, ω_1 and ω_2, are centered at O_1 and O_2 respectively. The circles ω_1 and ω_2 are externally tangent to each other and internally tangent to a larger circle ω centered at O at points A and B, respectively. Let M be the midpoint of minor arc AB. Let P be the intersection of ω_1 and O_1M, and let Q be the intersection of ω_2 and O_2M. Given that there is a point R on ω such that $\triangle PQR$ is equilateral, the radius of ω can be written as $\frac{a+\sqrt{b}}{c}$ where a, b, and c are positive integers and a and c are relatively prime. Find $a+b+c$.

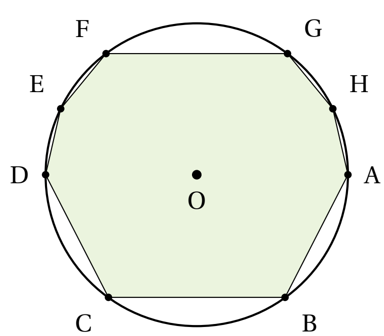
4. **[12]** Zandrew Hao has n^2 dollars, where n is an integer. He is a massive fan of the singer Pachary Zerry, and he wants to buy many copies of his 3 albums, which cost \$8, \$623, and \$835 (two of them are very rare). Find the sum of the 3 greatest values of n such that Zandrew can't spend all of his money on albums.

5. **[14]** In a rectangular prism with volume 24, the sum of the lengths of its 12 edges is 60, and the length of each space diagonal is $\sqrt{109}$. Let the dimensions of the prism be $a \times b \times c$, such that $a > b > c$. Given that a can be written as $\frac{p+\sqrt{q}}{r}$ where p, q, and r are integers and q is square-free, find $p+q+r$.

6. **[16]** Jared has 3 distinguishable Rolexes. Each day, he selects a subset of his Rolexes and wears them on his arm (the order he wears them does not matter). However, he does not want to wear the same Rolex 2 days in a row. How many ways can he wear his Rolexes during a 6 day period?

7. **[18]** Find the number of ways to tile a 12×3 board with 1×4 and 2×2 tiles with no overlap or uncovered space.

8. **[20]** Octagon $ABCDEFGH$ is inscribed in a circle where $AB = BC = CD = FG = 13$ and $DE = EF = GH = HA = 5$. The area of $ABCDEFGH$ can be expressed as $a + b\sqrt{c}$ where a, b, and c are positive integers, $\gcd(a,b) = 1$, and c is squarefree. Find $a+b+c$.



9. **[22]** There exist some number of ordered triples of real numbers (x, y, z) that satisfy the following system of equations:

$$x + y + 2z = 6$$
$$x^2 + y^2 + 2z^2 = 18$$
$$x^3 + y^3 + 2z^3 = 54$$

Given that the sum of all possible positive values of x can be expressed as $\frac{a+b\sqrt{c}}{d}$ where a,b,c, and d are positive integers, c is squarefree, and $\gcd(a, b, d) = 1$, find the value of $a + b + c + d$.

10. **[24]** Convex cyclic quadrilateral $ABCD$ satisfies $AC \perp BD$ and AC intersects BD at H. Let the line through H perpendicular to AD and the line through H perpendicular to AB intersect CB and CD at P and Q, respectively. The circumcircle of $\triangle CPQ$ intersects line AC again at $X \neq C$. Given that $AB = 13$, $BD = 14$, and $AD = 15$, the length of AX can be written as $\frac{a}{b}$ where a and b are relatively prime positive integers. Find $a + b$.

11. **[TIEBREAKER]** Estimate the value of e^f, where $f = e^e$.